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ROBUST CONTROLLER DESIGNS FOR SECOND-ORDER DYNAMIC SYSTEMS: A VIRTUAL PASSIVE APPROACH

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ABSTRACT

This paper presents a robust controller design for second-order dynamic systems. The controller is model-independent and itself is a virtual second-order dynamic system. Conditions on actuator and sensor placements are identified for controller designs that guarantee overall closed-loop stability. The dynamic controller can be viewed as a virtual passive damping system that serves to stabilize the actual dynamic system. The control gains are interpreted as virtual mass, spring, and dashpot elements that play the same roles as actual physical elements in stability analysis. Position, velocity, and acceleration feedback are considered. Simple examples are provided to illustrate the controller design. From this illustration, the physical meaning of the controller design is apparent.

INTRODUCTION

Control theory for time-invariant linear systems which are described by first-order dynamic equations have been well established for decades. Control software tools today are also written in first-order forms. For applications, engineers can simply convert whatever models they have to the first-order forms and then use the existing tools to design the controllers. If the performance requirements are satisfied by the controllers, the design jobs are completed. If not, the design parameters are changed and the design procedure continues until a satisfactory design is found. For a small scale system, a few design iterations may be enough to complete a satisfactory design. However, for a large scale system such as the space station, the dynamic model usually involves a large number of degrees of freedom and is best described by second-order dynamic equations in terms of sparse structural matrices including mass and stiffness matrices. For second-order dynamic systems, transforming to first-order form not only increases the dimension of the problem, but also destroys the sparsity of the structural matrices, i.e. the mass and stiffness matrices for flexible structures. As a result, computational efficiency and physical insight are lost in the first-order form. Existing control analysis and design software may not be able to handle such a large system due to computational difficulties. For example, solving a 1000-by-1000-dimension Riccati equation is considered numerically impossible using today's numerical techniques. There are basically two ways to address the controller design problems for a large-scale system. One way is to minimize the dimension of the system model by first preserving the second-order form and then performing model reduction. Laboratory experiments are required to verify the reduced model for robust controller designs. Recently controller designs using second-order system equations directly have gained attention in the literature as identified in Ref. [1]. Their computational advantages and physical features are also illustrated in Refs. [2] and [3]. Another way is to design a model-independent controller, which is insensitive to system uncertainties. The objective of this paper is to derive model-independent controllers for dynamic systems using second-order dynamic equations.

When a mass-spring-dashpot is attached to any mechanical system, including flexible space structures, the damping of the system is almost always augmented regardless of the system size. The parameters of the mass-spring-dashpot are arbitrary, model-independent and thus insensitive to the system uncertainties. To satisfy the system performance requirements, the parameters are adjusted using the knowledge of the system model. The more the system is known, the better the parameters of the mass-spring-dashpot may be adjusted to meet the performance requirements. However, no matter what happens, the mass-spring-dashpot won't destabilize the system because it is an energy-dissipative device. The question arises as to if there are any feedback control designs using sensors and actuators which behave like the passive mass-spring-dashpot. This paper is motivated by this question and the answer is very encouraging.

A novel approach for control of flexible structures is developed using a controller which can be described by a set of second-order dynamic equations. Under certain realistic (practical) conditions, this method provides a stable system with an infinite gain margin. For better understanding, two major steps are involved in developing the formulation of the method. First, consider only the direct output feedback for simplicity, implying the absence of dynamics in the feedback controller. Conditions are identified in terms of the number and type of sensors and their locations to make the system asymptotically stable with an infinite gain margin. Second, assume that the feedback controller contains a set of second-order dynamic equations. It is equivalent to visualize an imaginary flexible body, i.e. the feedback controller, which is linked side by side to the real flexible body. In other words, two sets of second-order dynamic equations are coupled to generate a closed-loop system. Design freedom increases when the dimension of the controller dynamic equations increases. Conditions are derived for the design of a stable closed-loop system having an infinite gain margin. The method takes advantage of the second-order form of equations (instead of transforming to a first-order form) which provides an easy way of discussing and obtaining the stability margin and results in a considerable computational efficiency for numerical simulations. Comparisons between the active feedback and the passive mass-spring-dashpot are given through several illustrative examples.

DIRECT FEEDBACK

In the analysis and design of dynamics and vibration control of flexible structures, two sets of linear, constant coefficient, ordinary differential equations are frequently used

$$M\ddot{x} + D\dot{x} + Kx = Bu \tag{1}$$

$$y = H_a\ddot{x} + H_v\dot{x} + H_d\, x \tag{2}$$

Here x is an $n \times 1$ state vector, and M, D, and K are mass, damping and stiffness matrices, respectively, which generally are symmetric and sparse. The $n \times p$ influence matrix B describes the actuator force distributions for the $p \times 1$ control force vector u. Typically, matrix M is positive definite whereas D and K are positive semi-definite. In the absence of rigid-body motion, K is positive definite. Equation (2) is a measurement equation having y as the $m \times 1$ measurement vector, H_a the $m \times n$ acceleration influence matrix, H_v the $m \times n$ velocity influence matrix, and H_d the $m \times n$ displacement influence matrix. Note that Eq. (1) can be solved for the acceleration in terms of the displacement, velocity and control force to obtain a new measurement equation in place of Eq. (2). However, physical insight is lost in this approach to controller design.

The measurement equation, Eq. (2), may be used either directly or indirectly for a feedback controller design. Here we will use direct feedback. Let the input vector u be

$$u = -Gy = -GH_a\ddot{x} - GH_v\dot{x} - GH_d x \tag{3}$$

where G is a gain matrix to be determined. Substituting Eq. (3) into Eq. (1) yields

$$(M + BGH_a)\ddot{x} + (D + BGH_v)\dot{x} + (K + BGH_d)x = 0 \tag{4}$$

For simplicity, consider the case where $H_a = H_d = 0$. Assume that the number of sensors m is larger than the number of actuators p. Let the actuators be located such that the row space generated by B^T belongs to the row space generated by H_v. In other words, the actuators are located in such a way that the control influence matrix B can be expressed by

$$B^T = C_b H_v \tag{5}$$

where C_b is a $p \times m$ matrix which may be obtained by $C_b = B^T H_v^T (H_v H_v^T)^{-1}$. Assume that the gain matrix G is computed by

$$G = LL^T C_b \tag{6}$$

where L is a $p \times p$ arbitrary matrix. Substituting Eq. (6) in Eq. (4) and noting the assumption that $H_d = 0$ leads to

$$M\ddot{x} + (D + BLL^T B^T)\dot{x} + Kx = 0 \tag{7}$$

For the case where $p < m < n$, BLL^TB^T is positive semi-definite and thus $D + BLL^TB^T$ is at least positive semi-definite for a positive semi-definite matrix D. As a result, the closed-loop system, Eq. (7), is stable if $D + BLL^TB^T$ is positive semi-definite, or asymptotically stable if $D + BLL^TB^T$ is positive definite. For the case where D is positive definite, $D + BLL^TB^T$ is positive definite which yields an asymptotically stable closed-loop system. This leads to a conclusion that, for a structural system with some passive damping, an output velocity feedback scheme with non-collocated velocity sensors and actuators may make the closed-loop system asymptotically stable with an *infinite gain margin* since L in Eq. (6) for determination of the gain matrix G is an arbitrary matrix, as long as the actuators are properly located satisfying Eq. (6). Note that, for colocated sensors and actuators, $B^T = H_v$. Without velocity measurements, the system damping cannot be augmented from direct output feedback alone. However, if there are actuator dynamics involved, the system damping may be augmented by direct displacement or acceleration feedback. See Ref. [3].

CONTROLLER WITH SECOND-ORDER DYNAMICS

Assume that the controller to be designed has a set of second-order dynamic equations and measurement equations similar to the system equations, Eqs. (1) and (2)

$$M_c \ddot{x}_c + D_c \dot{x}_c + K_c x_c = B_c u_c \tag{8}$$

$$y_c = H_{ac} \ddot{x}_c + H_{vc} \dot{x}_c + H_{dc} x_c \tag{9}$$

Note that this is a set of imaginary equations which do not represent any physical system. In fact, this set of equations basically serves as a filter to shift the phase of measurement signals. Here x_c is the controller state vector of dimension n_c, and M_c, D_c, and K_c are thought of as the controller mass, damping, and stiffness matrices, respectively, which generally are symmetric and positive definite to make the controller asymptotically stable. The $n_c \times m$ influence matrix B_c describes the force distributions for the $m \times 1$ input force vector u_c. Equation (9) is the controller measurement equation having y_c as the measurement vector of length p, H_{ac} the $p \times n_c$ acceleration influence matrix, H_{vc} the $p \times n_c$ velocity influence matrix and H_{dc} the $p \times n_c$ displacement influence matrix. Again, all the quantities, u_c, y_c, and n_c are imaginary and thus arbitrary which means that M_c, D_c, K_c, H_{dc} and H_{vc} are the design parameters for the controller.

Let the input vectors u, u_c in Eq. (1) and in Eq. (8) be

$$u = y_c = H_{ac} \ddot{x}_c + H_{vc} \dot{x}_c + H_{dc} x_c \tag{10}$$

$$u_c = y = H_a \ddot{x} + H_v \dot{x} + H_d x \tag{11}$$

Substituting Eq. (10) into Eq. (1) and Eq. (11) into Eq. (8) yields

$$M_t \ddot{x}_t + D_t \dot{x}_t + K_t x_t = 0 \tag{12}$$

where

$$M_t = \begin{bmatrix} M & -BH_{ac} \\ -B_c H_a & M_t \end{bmatrix}, D_t = \begin{bmatrix} D & -BH_{vc} \\ -B_c H_v & D_c \end{bmatrix}, K_t = \begin{bmatrix} K & -BH_{dc} \\ -B_c H_d & K_c \end{bmatrix}, x_t = \begin{bmatrix} x \\ x_c \end{bmatrix}$$

If the design parameters, M_c, D_c, K_c, H_{dc} and H_{vc} are chosen such that M_t, D_t and K_t are positive definite, the closed-loop system, Eq. (12), becomes asymptotically stable.

DISPLACEMENT FEEDBACK

For better understanding of the advantage of the controller having second-order dynamic equations, consider a special case where $H_a = H_{ac} = H_v = H_{vc} = 0$. To make K_t symmetric, it is required that

$$BH_{dc} = H_d^T B_c^T \tag{13}$$

or

$$\begin{bmatrix} B & -H_d^T \end{bmatrix} \begin{bmatrix} H_{dc} \\ B_c^T \end{bmatrix} = 0 \tag{14}$$

For the case where the sum of the number of actuators, p, and the number of sensors, m, is less than the number of states, n, the left-most matrix of Eq. (14) is a tall matrix. Unless B is in the space spanned by H_d^T or vice versa, there does not exist any solutions for B_c, H_{dc} in Eq. (13). Assume that the number of sensors, m, is larger than the number of actuators, p. Let the actuators be located such that the row space generated by B^T belongs to the row space generated by H_d, i.e. the actuators are located in such a way that the control influence matrix B can be expressed by

$$B^T = Q_b H_d \tag{15}$$

where Q_b is a $p \times m$ matrix which may be obtained by $Q_b = B^T H_d^T (H_d H_d^T)^{-1}$. Substituting Eq. (15) into Eq. (13) yields

$$H_d^T Q_b^T H_{dc} = H_d^T B_c^T \tag{16}$$

Since H_d^T is a tall matrix for $m < n$, the only possible solution is

$$Q_b^T H_{dc} = B_c^T \tag{17}$$

For any given matrix H_{dc}, this equation produces a B_c^T which makes the matrix K_t symmetric, i.e.,

$$K_t = \begin{bmatrix} K & -BH_{dc} \\ -H_{dc}^T B^T & K_c \end{bmatrix} \quad or \quad K_t = \begin{bmatrix} K & -H_d^T B_c^T \\ -B_c H_d & K_c \end{bmatrix} \tag{18}$$

The next question is how to choose a matrix H_{dc} which makes the closed-loop stiffness matrix K_t positive definite. The matrix K_t is positive definite, generally written as $K_t > 0$, if and only if

$$x_t^T K_t x_t > 0 \tag{19}$$

for any real vector x_t except the null vector. Substituting the definition of K_t and x_t from Eq. (12) in Eq. (19) yields

$$\begin{aligned} x_t^T K_t x_t &= x^T(K - H_d^T B_c^T B_c H_d)x + (B_c H_d x - x_c)^T (B_c H_d x - x_c) + x_c^T (K_c - I)x_c \\ &= x^T(K - B H_{dc} H_{dc}^T B^T)x + (H_{dc}^T B^T x - x_c)^T (H_{dc}^T B^T x - x_c) + x_c^T (K_c - I)x_c \end{aligned} \tag{20}$$

This equation is greater than zero if B_c and K_c are chosen such that $K - BH_{dc}H_{dc}{}^TB^T$ and $K_c - I$ are positive definite. Note that this is a sufficient condition but not a necessary condition. To make Eq. (19) hold, K must be a positive definite matrix, i.e. $K > 0$, and B_c must be chosen such that $K - BH_{dc}H_{dc}{}^TB^T \geq 0$. It implies that this controller may not be able to control rigid body motion since K in this case is only a positive semi-definite matrix, $K \geq 0$. To release the constraint condition, $K - BH_{dc}H_{dc}{}^TB^T > 0$, K must be increased by at least $BH_{dc}H_{dc}{}^TB^T$. In other words, the system must be stiffened which can be achieved by adding displacement feedback.

Let the input force be

$$u = y_c - Gy = H_{dc} x_c - G H_d x \tag{21}$$

where G is a gain matrix to be determined. Note that the velocity feedback is not considered here. Substituting Eq. (21) into the system equation, Eq. (1), the closed-loop stiffness matrix, Eq. (18), becomes

$$K_t = \begin{bmatrix} K + BGH_d & -H_d^T B_c^T \\ -B_c H_d & K_c \end{bmatrix} \tag{22}$$

If G is chosen such that

$$G = H_{dc} B_c \tag{23}$$

which from Eq. (13) results in

$$BGH_d = BH_{dc}B_cH_d = H_d^TB_c^TB_cH_d$$

The closed-loop stiffness matrix, Eq. (22), thus becomes

$$K_t = \begin{bmatrix} K + BH_{dc}H_{dc}^TB^T & -BH_{dc} \\ -H_{dc}^TB^T & K_c \end{bmatrix} \quad or \quad K_t = \begin{bmatrix} K + H_d^TB_c^TB_cH_d & -H_d^TB_c^T \\ -B_cH_d & K_c \end{bmatrix} \tag{24}$$

which changes Eq. (20) to be

$$\begin{aligned} x_t^TK_tx_t &= x^TKx + (B_cH_dx - x_c)^T(B_cH_dx - x_c) + x_c^T(K_c - I)x_c \\ &= x^TKx + (H_{dc}^TB^Tx - x_c)^T(H_{dc}^TB^Tx - x_c) + x_c^T(K_c - I)x_c \end{aligned} \tag{25}$$

Since K_c is a design parameter, the closed-loop system becomes stable as long as K_c is chosen larger than I, i.e. $x_c^T(K_c - I)x_c > 0$ for any arbitrary vector x_c. An obvious choice is $K_c = I$ where I is an identity matrix of dimension n_c. However, this is not the best choice which will be discussed later. To this end, it is shown that a stable closed-loop system can be designed using a feedback controller with second order dynamic equations. The controller has an infinite gain margin in the sense that the matrices M_c, D_c and K_c, which may be considered as the gain matrices for the controller state vector x_c and its derivatives, can be as large as desired without destabilizing the system as long as they are positive definite and K_c is larger than I.

A little modification of the above design produces a better design which has physical meaning. Indeed, let

$$B_c = K_c\overline{B}_c \quad or \quad \overline{B}_c = K_c^{-1}B_c \tag{26}$$

where K_c is assumed to be positive definite so that the solution for $\overline{B}_c$ exists for any given B_c. In addition, let the gain matrix G in Eq. (23) be slightly modified as follows

$$G = H_{dc}\overline{B}_c \tag{27}$$

which, with the aid of Eq. (13), results in

$$BGH_d = BH_{dc}\overline{B}_c H_d = H_d^T \overline{B}_c^T K_c \overline{B}_c H_d$$

The closed-loop stiffness matrix in this case (see Eq. (24)) thus becomes

$$K_t = \begin{bmatrix} K + H_d^T \overline{B}_c^T K_c \overline{B}_c H_d & -H_d^T \overline{B}_c^T K_c \\ -K_c \overline{B}_c H_d & K_c \end{bmatrix} \tag{28}$$

which in turn changes Eq. (25) to be

$$\begin{aligned} x_t^T K_t x_t &= x^T K x + (\overline{B}_c H_d x - x_c)^T K_c (\overline{B}_c H_d x - x_c) \\ &= x^T K x + (H_{dc}^T \overline{B}^T x - x_c)^T K_c (H_{dc}^T \overline{B}^T x - x_c) \end{aligned} \tag{29}$$

This equation is obviously positive if K is at least positive semi-definite, i.e. $K \geq 0$. Does this design have any physical meaning? The answer is positive. Consider the special case where the controller is as large as the system in the sense that the number of system states n is identical to the number of controller states n_c. Furthermore assume that all the states are directly measurable, $H_d = I$, and there are n actuators collocated with the sensors, $B = I$. In this case, $Q_b = I$ (Eq. (15)), $B_c = H_{dc} = K_c$ (Eq. (17)) for $\overline{B}_c = I$, and $G = K_c$ (Eq. (27)), which yields from Eq. (28)

$$K_t = \begin{bmatrix} K + K_c & -K_c \\ -K_c & K_c \end{bmatrix} \tag{30}$$

For a single degree of freedom $(n_c = n = 1)$, K_t represents the stiffness matrix for two springs connected in series with spring constants K and K_c.

PHYSICAL INTERPRETATION

For better understanding of the nature of the dynamic control designs developed here, they are now interpreted in physical terms. In this section, three illustrative examples will be shown, starting with a simple spring-mass system.

EXAMPLE 1: A simple spring-mass system with a single-degree-of-freedom controller

Consider a single-degree-of-freedom spring-mass system, $n_c = n = 1$, with displacement measurement of the system mass. The second-order controller for this case reduces to a virtual spring-mass-dashpot system connected in series with the system mass as shown in the following sketch

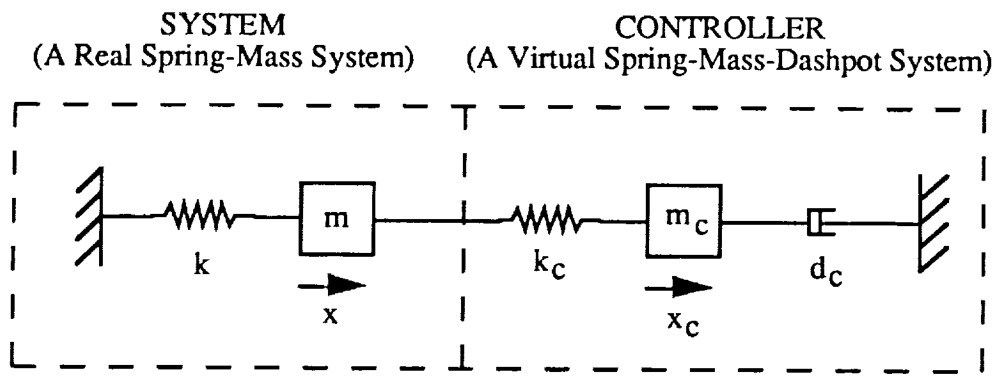

Fig. 1 A simple spring-mass system with a single-degree-of-freedom dynamic controller

Let the position of masses m and m_c be measured form their equilibrium states. The equations of motion for the above system can be derived by applying a force to m and m_c. The force applied to the system mass m in this case is the force F transmitted through the spring k_c. This is precisely the control force applied to the system as given in Eq. (21) with $H_{dc} = k_c$, $H_d = 1$, and $G = k_c$. Thus the second-order control law is simply

$$u = F = k_c\,(x_c - x) \tag{31}$$

where x_c is computed from

$$m_c\ddot{x}_c + d_c\dot{x}_c + k_c x_c = k_c x$$

The equation of motion which describes the closed-loop behavior of the above system is simply

$$\begin{bmatrix} m & 0 \\ 0 & m_c \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{x}_c \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & d_c \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} + \begin{bmatrix} k + k_c & -k_c \\ -k_c & k_c \end{bmatrix}\begin{bmatrix} x \\ x_c \end{bmatrix} = 0 \tag{32}$$

The above equation verifies Eq. (12) with $H_{vc} = H_v = H_{ac} = H_a = 0$ (i.e. no velocity and acceleration measurements), and K_t given in Eq. (30). The above set of equations is always stable for any m, k, m_c, k_c, and d_c, and is asymptotically stable for any $d_c > 0$. We now consider various special cases.

Case 1: For the controller without damping, $d_c = 0$, the system reduces to two spring-masses connected in series. If k_c is small, the control force given in Eq. (31) is small, thus the controller exerts little influence on the system. Mathematically, Eq. (32) becomes a set of two uncoupled equations of x and x_c, and obviously little change in the response of the controlled system is expected from this controller. If, however, k_c is large, (i.e. the virtual spring is stiff) the relative displacement between the two masses is small. Hence in the limit the two masses move together like a single mass $m + m_c$, and the natural frequency of the system is approaching

$$\omega_n = \sqrt{\frac{k}{m + m_c}} \tag{33}$$

As a result, for large k_c, changing the design variable m_c will affect the natural frequency of the closed-loop system according to Eq. (33) above.

Case 2: For $d_c > 0$, the system is always asymptotically stable (unless $k_c = 0$, which as discussed before means no control). The energy flows from m to m_c and is dissipated by the damper. Again, for large k_c, the system can be approximated as

$$(m + m_c)\ddot{x} + d_c\dot{x} + kx = 0 \tag{34}$$

Introduce the notation

$$\frac{d_c}{m + m_c} = 2\zeta\omega_n \quad , \quad \frac{k}{m + m_c} = \omega_n^2$$

Thus,

$$\zeta = \frac{1}{2}\frac{d_c}{\sqrt{k(m + m_c)}} \tag{35}$$

The design variables in this case are d_c and m_c. Various choices of d_c and m_c will result in $\zeta > 1$, $\zeta < 1$, or $\zeta = 1$, which corresponds to the cases the closed-loop system is over-damped, under-damped, or critically damped, respectively.

Case 3: For general values of k_c, d_c, and m_c, the design can be thought of as a virtual vibration absorber. Let the system be excited by some unknown force $Fe^{j\omega_f t}$, and the displacement of the mass m be denoted by $x = Xe^{j(\omega t + \phi)}$. The typical objective of a vibration absorber design is to determine the values of k_c, d_c, and m_c such that the ratio

$$\gamma = \left| \frac{Xe^{j(\omega t + \phi)}}{Fe^{j\omega_f t}} \right|$$

is minimized over an interested range of excitation frequency ω_f.

EXAMPLE 2: A two-degree-of-freedom system with a single-degree-of-freedom dynamic controller
Next, consider a two-degree of freedom spring-mass system with displacement measurements of the masses m_1 and m_2 from their equilibrium positions, $n = 2$. First consider the case where the the controller has only one state, $n_c = 1$. The second order controller in this case is simply equivalent to a virtual spring-mass-dashpot system connected in series with the two system masses as shown below

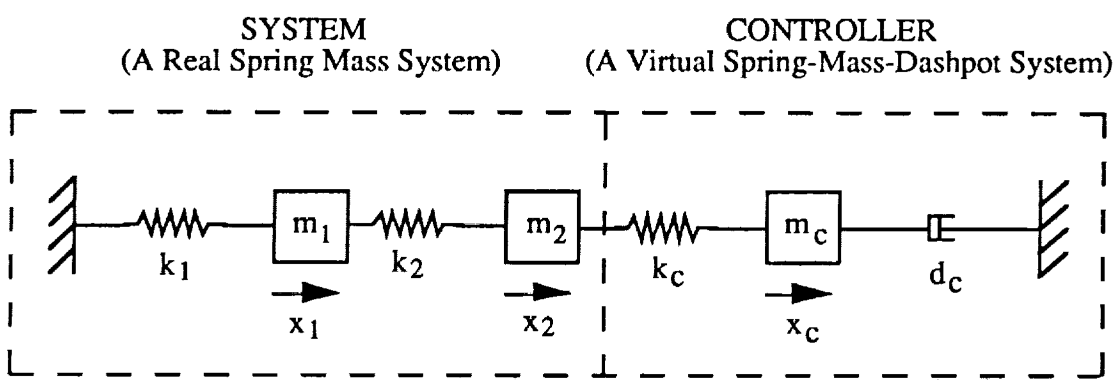

Fig. 2 A two-degree of freedom system with a single-degree-of-freedom dynamic controller

It can be easily shown that the control force applied to the system is simply

$$u = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0 \\ k_c\,(x_c - x_2) \end{bmatrix} \tag{36}$$

where F_j denotes the force applied to m_j, $j = 1, 2$; and x_c is given by

$$m_c\ddot{x}_c + d_c\dot{x}_c + k_c x_c = k_c x_2$$

Furthermore, the closed-loop behavior of the above system is governed by

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_c \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_c \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_c \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_c & -k_c \\ 0 & -k_c & k_c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_c \end{bmatrix} = 0 \tag{37}$$

which again verifies Eq. (12) with $H_{vc} = H_v = H_{ac} = H_a = 0$ and with K_t given in Eq. (30). Note that the above scheme requires only displacement measurement of the mass m_2.

EXAMPLE 3: A two-degree-of-freedom system with a two-degree-of-freedom dynamic controller

Consider the two-degree-of-freedom system above again with displacement measurements only, but now displacement measurement of the mass m_1 is also to be used in the controller design. The second order controller design in this case is simply

$$u = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} k_{c_1} & 0 \\ 0 & k_{c_2} \end{bmatrix} \begin{bmatrix} x_{c_1} - x_1 \\ x_{c_2} - x_2 \end{bmatrix} \tag{38}$$

where

$$\begin{bmatrix} m_{c_1} & 0 \\ 0 & m_{c_2} \end{bmatrix} \begin{bmatrix} \ddot{x}_{c_1} \\ \ddot{x}_{c_2} \end{bmatrix} + \begin{bmatrix} d_{c_1} & 0 \\ 0 & d_{c_2} \end{bmatrix} \begin{bmatrix} \dot{x}_{c_1} \\ \dot{x}_{c_2} \end{bmatrix} + \begin{bmatrix} k_{c_1} & 0 \\ 0 & k_{c_2} \end{bmatrix} \begin{bmatrix} x_{c_1} \\ x_{c_2} \end{bmatrix} = \begin{bmatrix} k_{c_1} & 0 \\ 0 & k_{c_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{39}$$

The closed-loop system is equivalent to a mass-spring-dashpot system shown below

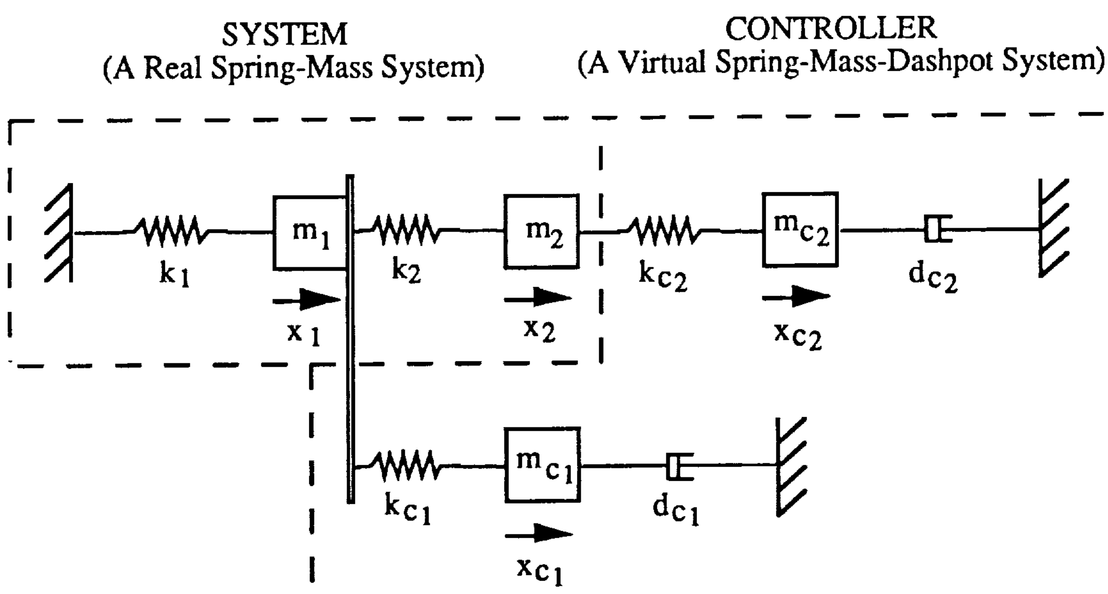

Fig. 3 A two-degree of freedom system with a two-degree-of-freedom dynamic controller

whose behavior is governed by

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_{c_1} & 0 \\ 0 & 0 & 0 & m_{c_2} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_{c_1} \\ \ddot{x}_{c_2} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & d_{c_1} & 0 \\ 0 & 0 & 0 & d_{c_2} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{c_1} \\ \dot{x}_{c_2} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 + k_{c_1} & -k_2 & -k_{c_1} & 0 \\ -k_2 & k_2 + k_{c_2} & 0 & -k_{c_2} \\ -k_{c_1} & 0 & k_{c_1} & 0 \\ 0 & -k_{c_2} & 0 & k_{c_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_{c_1} \\ x_{c_2} \end{bmatrix} = 0 \qquad (40)$$

If velocity measurements are available, say at the system mass m_1, then a dashpot element may be added in between m_1 and m_{c1} for example. It should be noted, however, that the controller masses, springs, dashpots are in fact virtual elements with physical interpretations as such. For ground based systems, they may represent actual physical elements attached to the ground. But for space based systems, they are simply controller gains in the control algorithm.

ACCELERATION FEEDBACK

The above controller can be extended to acceleration feedback as well. Consider the system given in Eq. (1), but now the measurement vector y in Eq. (2) has only acceleration measurements, i.e. $H_v = H_d = H_{vc} = H_{dc} = 0$ in Eq. (12)

$$M_t \ddot{x}_t + D_t \dot{x}_t + K_t x_t = 0$$

where

$$M_t = \begin{bmatrix} M & -BH_{ac} \\ -B_c H_a & M_c \end{bmatrix}, \quad D_t = \begin{bmatrix} D & 0 \\ 0 & D_c \end{bmatrix}, \quad K_t = \begin{bmatrix} K & 0 \\ 0 & K_c \end{bmatrix}$$

To make M_t symmetric, it is required that $BH_{ac} = H_a^T B_c^T$ as discussed in Eqs. (13)-(17). All the discussions regarding the positive-definiteness of K_t from Eqs. (18)-(20) also apply to M_t.

Additional coupling in the closed-loop mass matrix M_t can be achieved by letting the input u in Eq. (12) include direct acceleration feedback, i.e.,

$$u = y_c - G_a y = H_{ac} \ddot{x}_c - G_a y \qquad (41)$$

which makes M_t become

$$M_t = \begin{bmatrix} M + BGH_a & -BH_{ac} \\ -B_c H_a & M_c \end{bmatrix} \qquad (42)$$

As before, M_t can be made symmetric and positive definite by proper choices of H_{ac}, B_c, and G_a.

Let

$$B_c = M_c\overline{B}_c \quad or \quad \overline{B}_c = M_c^{-1}B_c \tag{43}$$

where M_c is positive definite so that the solution to $\overline{B}_c$ exists for any given B_c. Let G be chosen such that

$$G = H_{ac}\overline{B}_c = H_{ac}M_c^{-1}B_c \tag{44}$$

which, with the aid of the equality, $BH_{ac} = H_a{}^TB_c{}^T$, results in

$$BGH_a = BH_{ac}\overline{B}_cH_a = H_a^TB_c^T\overline{B}_cH_a = H_a^T\overline{B}_c^TM_c\overline{B}_cH_a \tag{45}$$

The closed-loop mass matrix in this case becomes

$$M_t = \begin{bmatrix} M + H_a^T\overline{B}_c^TM_c\overline{B}_cH_a & -H_a^T\overline{B}_c^TM_c \\ -M_c\overline{B}_cH_a & M_c \end{bmatrix} \tag{46}$$

This is a positive definite matrix as discussed in Eq. (29) for K_t, regardless of the value of M as long as M is positive definite. The closed-loop in this case becomes

$$\begin{bmatrix} M + H_a^T\overline{B}_c^TM_c\overline{B}_cH_a & -H_a^T\overline{B}_c^TM_c \\ -M_c\overline{B}_cH_a & M_c \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{x}_c \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & D_c \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K_c \end{bmatrix}\begin{bmatrix} x \\ x_c \end{bmatrix} = 0 \tag{47}$$

Figure 4 shows a block diagram of the closed-loop system with acceleration feedback. In order to have a different flavor for the readers, all the quantities are expressed in frequency domain to quantify the multi-variable stability margins and performance of such systems. Let $G_s(s) = H_a[Ms^2 + Ds + K]^{-1}B$ be the system transfer function, $G_c(s) = H_{ac}[M_cs^2 + D_cs + K_c]^{-1}B_c$ the controller transfer function, and $G_a = H_{ac}M_c^{-1}B_c$ the direct acceleration feedback gain. The acceleration measurement $y(s)$ caused by the application of an external force $r(s)$ can be expressed by

$$y(s) = s^2G_s(s)\left[r(s) + \left(s^2G_c(s) - G_a\right)y(s)\right] \tag{48}$$

or

$$y(s) = \left[I - s^2 G_s(s)\left(s^2 G_c(s) - G_a\right)\right]^{-1} s^2\, G_s(s) r(s)$$

The closed-loop transfer function from $r(s)$ to $y(s)$ is

$$G(s) = \left[I - s^2 G_s(s)\left(s^2 G_c(s) - G_a\right)\right]^{-1} s^2\, G_s(s) \tag{49}$$

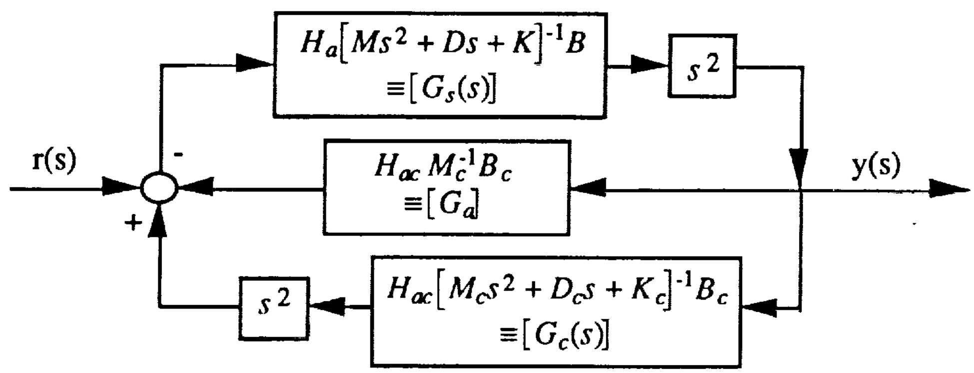

Fig. 4 Block diagram of the closed-loop system with acceleration feedback

It is interesting to note that

$$s^2 G_c(s) - G_a = -H_{ac}\left(M_c s^2 + D_c s + K_c\right)^{-1}\left(D_c s + K_c\right) M_c^{-1} B_c \tag{50}$$

Figure 5 is equivalent to Fig. 4 for the closed-loop system with acceleration feedback. All the quantities M_c, D_c, and K_c are design parameters which are model independent but they must be positive definite. The quantities H_{ac} and B_c are related by H_a and B such that $BH_{ac} = H_a^T B_c^T$. This system is always stable regardless of how much uncertainties occur in the system matrices M, D, and K.

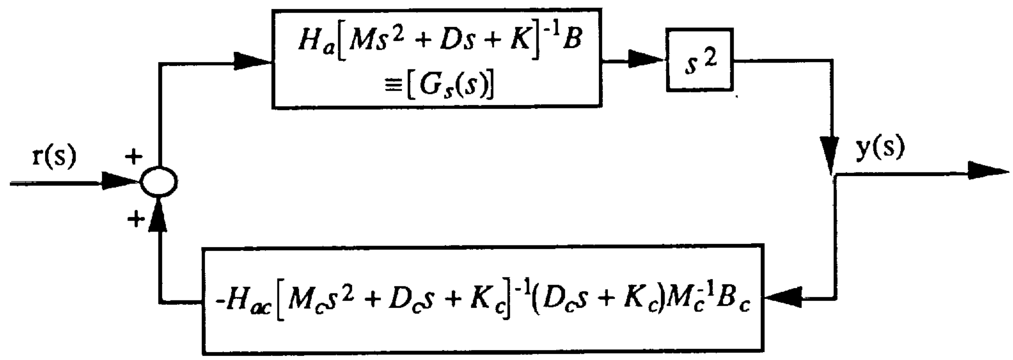

Fig. 5 Simplified block diagram of the closed-loop system with acceleration feedback

Let $\overline{H}_{ac}$ be $\overline{H}_{ac} = H_{ac} M_c$, and recall that $B_c = M_c \overline{B}_c$. Equation (50) becomes

$$s^2G_c(s) - G_a = -\overline{H}_{ac}\left(s^2 + M_c^{-1}D_cs + M_c^{-1}K_c\right)^{-1}(D_cs + K_c)\overline{B}_c \tag{51}$$

For the case where M_c is sufficiently large such that $M_c^{-1}D_c$ and $M_c^{-1}K_c$ may be neglected, the above equation can be approximated by

$$s^2G_c(s) - G_a \approx -\overline{H}_{ac}(D_cs + K_c)\overline{B}_cs^{-2} \tag{52}$$

Figure 5 can then be reduced to yield Fig. 6. For the case $K_c = 0$, the controller becomes an integrator of the acceleration measurement. If H_{ac} is chosen to be B_c^T, then

$$s^2G_c(s) - G_a \approx -\overline{B}_c^TD_c\overline{B}_c s^{-1}$$

which is equivalent to a direct velocity feedback to the system.

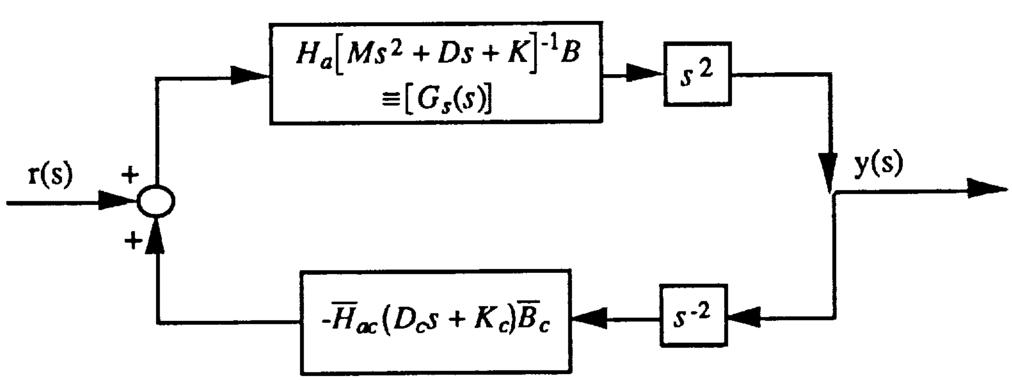

Fig. 6 Reduced block diagram of the closed-loop system for a sufficiently large Mc

Although it seems logical to choose a large mass matrix M_c for the controller, measurement bias and noises may prevent such a choice in practice, because integrating a bias is obviously not desirable in a control loop.

The procedure for deriving the second-order controller with acceleration feedback is identical to that for displacement feedback. Mathematically, both controllers are identical in the sense that the closed-loop mass matrix M_t for acceleration feedback can be obtained by replacing K by M in the closed-loop stiffness matrix K_t for displacement feedback, and subscript d by a. In other words, both displacement and acceleration feedback are conceptually dual. However, significant differences between both controllers appear when they are implemented either actively, or passively, which will be shown in the following example.

EXAMPLE 4: A single-degree-of-freedom system with acceleration feedback

Consider a single degree-of-freedom spring-mass system with acceleration measurement of the system mass. The second-order controller for this case reduces to a virtual spring-mass-dashpot connected in series with the system mass as shown in Fig. 7

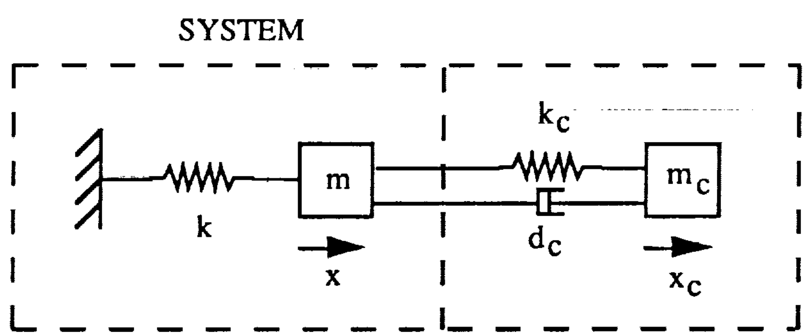

Fig. 7 A simple spring-mass system with acceleration feedback

Note that the vector x_c here means the relative position of m_c to the position of m. In this case, H_a and $\overline{B}_c$ in Eq. (47) are chosen to be $\overline{B}_c = -H_a = 1$. The second-order control law is

$$u = -m_c\left(\ddot{x}_c + \ddot{x}\right)$$

where x_c is computed from

$$m_c \ddot{x}_c + d_c \dot{x}_c + k_c x_c = -m_c \ddot{x}$$

The closed-loop system can then be rewritten as

$$\begin{bmatrix} m + m_c & m_c \\ m_c & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{x}_c \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & d_c \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} = 0$$

The transfer function , Eq. (49), becomes

$$G(s) = \frac{s^2}{\left(ms^2 + k\right) + m_c s^2 (d_c s + k_c)\left(m_c s^2 + d_c s \;+ k_c\right)^{-1}}$$

For large m_c, $G(s)$ reduces to

$$G(s) \approx \frac{s^2}{ms^2 + d_c s \;+ (k + k_c)}$$

The system is clearly asymptotically stable. The numerator s^2 appears due to the acceleration feedback.

Comparison of Fig. 1 and Fig. 7 reveals the difference between the acceleration and displacement feedback controllers. The controller for acceleration feedback does not have a virtual ground attached to the control mass and thus cannot control the rigid body motion.

CONCLUSIONS

This paper formulates a robust second-order dynamic stabilization controller design for second-order dynamic systems. The design is passive in the sense that it contains mechanisms that serve only to transfer and dissipate energy of the system. The controller interacts with the physical system only through spring, mass, and dashpot elements, and therefore, it can be implemented actively or passively. In other words, stabilization can be accomplished either by a controller with gains interpreted as virtual mass, spring, and dashpot elements, or by actual physical masses, springs, and dashpots connected to the system.

The passive design means that the controller does not destabilize the system. As far as stability is concerned, the controller is model independent, and this is a robust design. Specifically, overall closed-loop stability is guaranteed independently of the system structural uncertainty and variations in the structural parameters. It should be emphasized that this is a robustness result with respect to structural uncertainty in the absence of measurement uncertainty and other contributing factors.

However, control performance, unlike stability robustness, is dependent on the system characteristics. Knowledge of the system model can always help improve a controller design. In this method, the controller order and/or controller gains can be adjusted to meet the desired performance. Physical interpretation of the controller gains as virtual masses, springs, and dashpots provides convenient rules of thumb as to how they should be adjusted to meet a certain desired performance objective.

Finally, the controller has been formulated from the continuous-time setting. Actual implementation of the controller, however, most likely requires usage of a digital computer. In future work, effects of sampling and time delays will be addressed. Other practical issues that can also affect the control performance such as measurement noises, actuator and sensor saturation limits will be investigated. It should also be interesting to examine the difference and relation between this approach and others such as full state feedback with a second-order state estimator.

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16. Abstract

This paper presents a robust controller design for secon-order dynamic systems. The controller is model-independent and itself is a virtual second-order dynamic system. Conditions on actuator and sensor placements are identified for controller designs that guarantee overall closed-loop stability. The dynamic controller can be viewed as a virtual passive damping system that serves to stabilize the actual dynamic system. The control gains are interpreted as virtual mass, spring, and dashpot elements that play the same roles as actual physical elements in stability analysis. Position, velocity, and acceleration feedback are considered. Simple examples are provided to illustrate the physical meaning of this controller design.

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